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B to light meson transition form factors calculated in perturbative QCD approach

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Abstract. We calculate the $B \to P$, $B \to V$ (P is the light pseudoscalar meson, V the light vector meson) form factors in the large-recoil limit in the perturbative QCD approach, including both the vector (axial vector) and tensor operators. In general there are two leading components ϕ_B and $\bar{\phi}_B$ for the B meson wave functions. We consider both contributions of them. Sudakov effects ($k_\perp$ and threshold resummation) are included to regulate the soft end-point singularity. By choosing the hard scale as the maximum virtualities of the internal particles in the hard b quark decay amplitudes, Sudakov factors can effectively suppress the long-distance soft contribution. The hard contribution can be dominant in these approaches.

1 Introduction

The most difficult task in calculating the B meson decay amplitude is to treat the hadronic matrix element $\langle M_1 M_2 | Q_i | B \rangle$, which is generally controlled by the soft non-perturbative dynamics of QCD. Here Q_i is one of the effective low energy transition operators of b quark decays [1], and M_1 and M_2 are the final state mesons produced in B decays. In the earlier years, these hadronic matrix elements of B decays were treated by an approximate method, which is called the factorization approach [2]. In the factorization approach the hadronic matrix element of the four-fermion operator is approximated as a product of the matrix elements of two currents, $\langle M_1 M_2 | Q_i | B \rangle \simeq \langle M_1 | j_{1\mu} | 0 \rangle \langle M_2 | j_2^\mu | B \rangle$, where $j_{1\mu}$ and j_2^μ are the two relevant currents which can be related to Q_i through $Q_i = j_{1\mu} j_2^\mu$. The matrix element of $j_{1\mu}$ sandwiched between the vacuum and meson state M_1 directly defines the decay constant of M_1. For example, if M_1 is a pseudoscalar and $j_{1\mu}$ is the $V{-}A$ current, the relation between the matrix element and the decay constant will be $\langle M_1 | j_{1\mu} | 0 \rangle = \mathrm{i} f_{M_1} p_\mu$, where f_{M_1} and p_μ are the decay constant and the four-momentum of M_1, respectively. The other matrix element $\langle M_2 | j_2^\mu | B \rangle$ can generally be decomposed into transition form factors of $B \to M_2$ due to its Lorentz property. The explicit definition of B meson transition form factors through such matrix elements can be found in Sects. 4 and 5.

In semi-leptonic decays of the B meson, the decay amplitude can be directly related to B meson transition form factors without the factorization approximation. For example, for $B \to \pi \ell \bar{\nu}_\ell$, the decay amplitude can be written in the form

$$\mathcal{A}(p_B, p_\pi) = \frac{G_\mathrm{F}}{\sqrt{2}} V_{ub} (\bar{\ell} \gamma_\mu (1 - \gamma_5) \nu_\ell) \times \langle \pi(p_\pi) | \bar{u} \gamma^\mu b | \bar{B}(p_B) \rangle, \quad (1)$$

where the form factors $F_1(q^2)$ and $F_0(q^2)$ are defined through the $B \to \pi$ transition matrix element $\langle \pi(p_\pi) | \bar{u} \gamma^\mu b | \bar{B}(p_B) \rangle$ in (14) of Sect. 4. In general the form factors are functions of the momentum transfer squared, $q^2 = (p_B - p_\pi)^2$. In the region of small recoil, where q^2 is large and/or the final particle is heavy enough, the form factors are dominated by soft dynamics, which is out of control of perturbative QCD. However, in the large-recoil region where $q^2 \to 0$, and when the final particle is light (such as the pion), 5 GeV ($m_B = 5$ GeV) of energy is released. About half of this energy is taken by the light final particle, which suggests that large momentum is transferred in this process and the interaction is mainly short-distance. Therefore perturbative QCD can be applied to B to light meson transition form factors in the large-recoil region.

Before applying the perturbative method in this calculation, one must separate soft dynamics from hard interactions. This is called factorization in QCD. The factorization theorem has been worked out in [3] based on the earlier work on the applications of perturbative QCD in hard exclusive processes [4], where the soft contributions are factorized into wave functions or distribution amplitudes of mesons, and the hard part is treated by perturbative QCD. Sudakov resummation has been introduced to suppress the long-distance contributions. Recently this

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approach has been well developed and extensively used to analyze B decays [5–13]. There is also another direction to prove factorization in the soft-collinear effective theory [14], which shows correctly the power counting rules in QCD. In this work we shall calculate a set of $B \to P$ and $B \to V$ (P is a light pseudoscalar meson, V a light vector meson) transition form factors in the perturbative QCD (PQCD) approach. We use the B wave functions derived in the heavy quark limit recently [15], and include Sudakov effects from transverse momentum $k_\perp$ and threshold resummation [9,16]. In general there are two Lorentz structures for the B wave functions. If they are appropriately defined, only one combination gives a large contribution; the other combination contributes 30%.

A direct calculation of the one-gluon-exchange diagram for the B meson transition form factors suffers from singularities from the end-point region of the light-cone distribution amplitude with a momentum fraction $x \to 0$ in the longitudinal direction. In fact, in the end-point region the parton transverse momenta $k_\perp$ are not negligible. After including the partons' transverse momenta, large double logarithmic corrections $\alpha_s \ln^2 k_\perp$ appear in higher order radiative corrections and have to be summed to all orders. In addition to the double logarithm like $\alpha_s \ln^2 k_\perp$, there are also large logarithms $\alpha_s \ln^2 x$ which should also be summed to all orders. This is called threshold resummation [16]. The relevant Sudakov factors from both $k_\perp$ and threshold resummation can cure the end-point singularity which makes the calculation of the hard amplitudes infrared safe. We check the perturbative behavior in the calculation of the B meson transition form factors and find that with the hard scale appropriately chosen, Sudakov effects can effectively suppress the soft dynamics, and the main contribution comes from the perturbative region.

The content of this paper is as follows. Section 2 treats the kinematics and the framework of the PQCD approach used in the calculation of $B \to P$ and $B \to V$ transition form factors. Section 3 includes wave functions of the B meson and the light pseudoscalar and vector mesons. We give the results of the $B \to P$ transition form factors in Sect. 4, and the $B \to V$ transition form factors in Sect. 5. Section 6 are the numerical results and discussion. Finally Sect. 7 is a brief summary.

2 The framework

Here we first give our conventions on kinematics. In light-cone coordinates, the momentum is taken in the form $k = \left(\frac{k^+}{\sqrt{2}}, \frac{k^-}{\sqrt{2}}, \mathbf{k}_\perp\right)$ with $k^\pm = k^0 \pm k^3$ and $\mathbf{k}_\perp = (k^1, k^2)$. The scalar product of two arbitrary vectors A and B is $A \cdot B = A_\mu B^\mu = \frac{1}{2}(A^+B^- + A^-B^+) - \mathbf{A}_\perp \cdot \mathbf{B}_\perp$. Our study is in the rest frame of the B meson. The mass difference of b quark and B meson is negligible in the heavy quark limit and we take $m_b \simeq m_B$ in our calculation. The masses of the light quarks u, d, s and the light pseudoscalar mesons are neglected, while the masses of the light vector mesons ρ, ω, K^* are kept in the first order. The momentum of light meson is chosen in the "+" direction. Under these conventions, the momentum of the B meson is $P_B = \frac{1}{\sqrt{2}}(m_B, m_B, \mathbf{0}_\perp)$, and in the large-recoil limit $q^2 \to 0$, the momentum of the light pseudoscalar meson is $P_P = \left(\frac{m_B}{\sqrt{2}}, 0, \mathbf{0}_\perp\right)$. For the case of the light vector meson, its momentum is $P_V = \frac{m_B}{\sqrt{2}}(1, r_V^2, \mathbf{0}_\perp)$ with r_V defined as $r_V \equiv m_V/m_B$. The longitudinal polarization of the vector meson is $\varepsilon_L = \frac{1}{\sqrt{2}}\left(\frac{1}{r_V}, -r_V, \mathbf{0}_\perp\right)$, its transverse polarization $\varepsilon_T = (0, 0, \mathbf{1}_\perp)$. The light spectator momenta k_1 in the B meson and k_2 in the light meson are parameterized as $k_1 = \left(0, x_1\frac{m_B}{\sqrt{2}}, k_{1\perp}\right)$ and $k_2 = \left(x_2\frac{m_B}{\sqrt{2}}, 0, k_{2\perp}\right)$, where k_2^- is dropped because of its smallness (In the meson moving along the "plus" direction with large momentum, the minus component of its parton's momentum k_2^- should be very small). We also dropped k_1^+ because it vanishes in the hard amplitudes, which can be simply shown below.

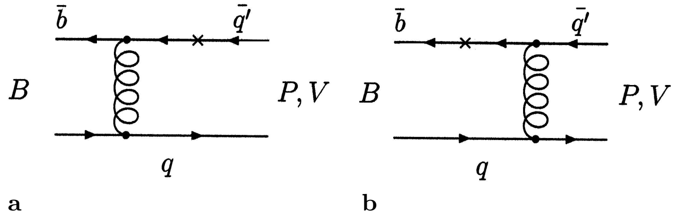

Fig. 1a,b. Diagrams contributing to the $B \to P, V$ form factors, where the cross denotes an appropriate gamma matrix

The lowest order diagrams for the B to light meson transition form factors are displayed in Fig. 1. The hard amplitudes H are proportional to the propagator of the gluon, i.e., $H \propto 1/(k_2 - k_1)^2 \simeq 1/(2k_2 \cdot k_1) \simeq 1/(k_2^+ k_1^-)$. It is obvious that only k_1^- is left in the hard amplitude.

Factorization is one of the most important parts of applying perturbative QCD in hard exclusive processes, which separates long-distance dynamics from short-distance dynamics. The factorization formula for the $B \to P, V$ transition matrix element can be written as

$$\begin{aligned}
&\langle P, V(P_2)|\bar{b}\Gamma_\mu q'|B(p_1)\rangle \\
&= \int \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}^2k_{1\perp} \mathrm{d}^2k_{2\perp} \frac{\mathrm{d}z^+\mathrm{d}^2z_\perp}{(2\pi)^3} \frac{\mathrm{d}y^+\mathrm{d}^2y_\perp}{(2\pi)^3} \\
&\times \mathrm{e}^{-\mathrm{i}k_2\cdot y} \langle P, V(P_2)|\bar{q}(y)_\alpha q'_\beta(0)|0\rangle H_\mu^{\beta\alpha;\sigma\rho} \mathrm{e}^{\mathrm{i}k_1\cdot z} \\
&\times \langle 0|\bar{b}(0)_\rho q_\sigma(z)|B(P_1)\rangle, \qquad (2)
\end{aligned}$$

where the matrix elements $\langle P, V(P_2)|\bar{q}(y)_\alpha q'_\beta(0)|0\rangle$ and $\langle 0|\bar{b}(0)_\rho q_\sigma(z)|B(P_1)\rangle$ define the wave functions of the light pseudoscalar (vector) meson and the B meson, which absorb all the soft dynamics. $H_\mu^{\beta\alpha;\sigma\rho}$ denotes the hard amplitude, which can be treated by perturbative QCD. β, α, σ and ρ are Dirac spinor indices. Both the wave functions and the hard amplitude H are scale dependent. This scale is usually called the factorization scale. Above this scale, the interaction is controlled by hard dynamics, while the interaction below this scale is controlled by soft dynamics, which is absorbed into wave functions. The factorization scale is usually taken to be the same as the renormalization scale. In practice it is convenient to work in transverse separation coordinate space (b-space) rather than the transverse momentum space ($k_\perp$-space). So we shall make a Fourier transformation $\int d^2k_\perp e^{-i\mathbf{k}_\perp \cdot \mathbf{b}}$ to transform the wave functions and hard amplitude into b-space. $1/b$ will appear as a typical factorization scale. As the scale $\mu > 1/b$, the interactions are controlled by the hard dynamics, and as $\mu < 1/b$ the soft dynamics dominates which is absorbed into the wave functions.

Higher order radiative corrections to wave functions and hard amplitudes generate large double logarithms through the overlap of collinear and soft divergences. The infrared divergence is absorbed into the wave functions. The double logarithms $\alpha_s \ln^2 Pb$ have been summed to all orders to give an exponential Sudakov factor $e^{-S(x,b,P)}$; here P is the typical momentum transferred in the relevant process, and x is the longitudinal momentum fraction carried by the relevant parton. The resummation procedure has been analyzed and the result has been given in [3]. We do not repeat it here.

In addition to double logarithms $\alpha_s \ln^2 Pb$ in b-space (or say $k_\perp$-space equivalently), radiative corrections to hard amplitudes also produce large logarithms $\alpha_s \ln^2 x$. These double logarithms should also be summed to all orders. This threshold resummation leads to [8]

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)}[x(1-x)]^c, \quad (3)$$

where the parameter $c = 0.3$. This function is normalized to unity. $S_t(x)$ vanishes very fast at the end-point region $x \to 0$ and $x \to 1$. Therefore the factors $S_t(x_1)$ and $S_t(x_2)$ suppress the end-point region of the meson distribution amplitudes.

3 The wave functions

In the resummation procedures, the B meson is treated as a heavy–light system. In general, the B meson light-cone matrix element can be decomposed as [17,18]

$$\begin{aligned}
&\int_0^1 \frac{d^4z}{(2\pi)^4} e^{i\mathbf{k}_1\cdot z}\langle 0|\bar{q}_\alpha(z)b_\beta(0)|\bar{B}(p_B)\rangle \quad (4)\\
&= \frac{i}{\sqrt{2N_c}}\left\{(\not p_B + m_B)\gamma_5\left[\frac{\not v}{\sqrt{2}}\phi_B^+(\mathbf{k}_1) + \frac{\not n}{\sqrt{2}}\phi_B^-(\mathbf{k}_1)\right]\right\}_{\beta\alpha}\\
&= -\frac{i}{\sqrt{2N_c}}\left\{(\not p_B + m_B)\gamma_5\left[\phi_B(\mathbf{k}_1) + \frac{\not n}{\sqrt{2}}\bar\phi_B(\mathbf{k}_1)\right]\right\}_{\beta\alpha},
\end{aligned}$$

where $n = (1,0,\mathbf{0}_T)$, and $v = (0,1,\mathbf{0}_T)$ are the unit vectors pointing to the plus and minus directions, respectively. From the above equation, one can see that there are two Lorentz structures in the B meson wave function. In general, one should consider both these two Lorentz structures in the calculations of the B meson decays. The light-cone distribution amplitudes ϕ_B^+ and ϕ_B^- are derived by Kawamura et al. in the heavy quark limit [15],

$$\begin{aligned}
\phi_B^+(x,b) &= \frac{f_B x}{\sqrt{6}\Lambda_0^2}\theta(\Lambda_0 - x)J_0\left[m_B b\sqrt{x(\Lambda_0 - x)}\right], \quad (5)\\
\phi_B^-(x,b) &= \frac{f_B(\Lambda_0 - x)}{\sqrt{6}\Lambda_0^2}\theta(\Lambda_0 - x)J_0\left[m_B b\sqrt{x(\Lambda_0 - x)}\right],
\end{aligned}$$

with $\Lambda_0 = 2\bar\Lambda/M_B$, and $\bar\Lambda$ is a free parameter which should be of the order of $m_B - m_b$.

The relations between ϕ_B, $\bar\phi_B$ and ϕ_B^+, ϕ_B^- are

$$\phi_B = \phi_B^+, \qquad \bar\phi_B = \phi_B^+ - \phi_B^-. \quad (6)$$

The normalization conditions for these two distribution amplitudes are

$$\int d^4k_1 \phi_B(\mathbf{k}_1) = \frac{f_B}{2\sqrt{2N_c}}, \quad \int d^4k_1 \bar\phi_B(\mathbf{k}_1) = 0. \quad (7)$$

From (5) and (6), we can see that when $x \to 0$, $\bar\phi_B \not\to 0$, while $\phi_B \to 0$. The behavior of ϕ_B with the definition (6) is similar to the one defined in previous PQCD calculations [5–13]. Note that our definitions of ϕ_B, $\bar\phi_B$ are different from the previous one in the literature [8]

$$\begin{aligned}
&\int_0^1 \frac{d^4z}{(2\pi)^4} e^{i\mathbf{k}_1\cdot z}\langle 0|\bar{q}_\alpha(z)b_\beta(0)|\bar{B}(p_B)\rangle = \frac{i}{\sqrt{2N_c}}\\
&\times\left\{(\not p_B + m_B)\gamma_5\left[\frac{\not v}{\sqrt{2}}\phi_B^+(\mathbf{k}_1) + \frac{\not n}{\sqrt{2}}\phi_B^-(\mathbf{k}_1)\right]\right\}_{\beta\alpha}\\
&= -\frac{i}{\sqrt{2N_c}} \quad (8)\\
&\times\left\{(\not p_B + m_B)\gamma_5\left[\phi_B'(\mathbf{k}_1) + \frac{\not n - \not v}{\sqrt{2}}\bar\phi_B'(\mathbf{k}_1)\right]\right\}_{\beta\alpha},
\end{aligned}$$

with

$$\phi_B' = \frac{\phi_B^+ + \phi_B^-}{2}, \qquad \bar\phi_B' = \frac{\phi_B^+ - \phi_B^-}{2}. \quad (9)$$

This definition is equivalent to (4) and (6) in the total amplitude. Although the final numerical results should be the same, the form factor formulas are simpler using our new definition. Another outcome is that our new formula shows explicitly the importance of the leading twist contribution ϕ_B, which will be shown later. However, if ϕ_B' and $\bar\phi_B'$ are defined as in (9), both of their contributions are equivalently important (see the numerical results in Table 2 of [19]). It is easy to check that both ϕ_B' and $\bar\phi_B'$ here have non-zero end-points at $x \to 0$. In this case, ϕ_B' does not correspond to the one defined in the previous PQCD calculations [8,9], where $\phi_B \to 0$, at the end-point, when $x \to 0$ or 1.

The π, K mesons are treated as a light–light system. At the B meson rest frame, the K meson (or pion) is moving very fast; one of k_1^+ or k_1^- is zero, depending on the definition of the z axis. We consider a kaon (or π meson) moving in the plus direction in this paper. The K meson distribution amplitude is defined by [20]

$$\begin{aligned}
&\langle K^-(P)|\bar{s_\alpha}(z)u_\beta(0)|0\rangle = \frac{i}{\sqrt{2N_c}}\int_0^1 dx e^{ixP\cdot z}\\
&\times\Big[\gamma_5 \not P\phi_K(x) + m_0\gamma_5\phi_P(x)\\
&\quad - m_0\sigma^{\mu\nu}\gamma_5 P_\mu z_\nu\frac{\phi_\sigma(x)}{6}\Big]_{\beta\alpha}. \quad (10)
\end{aligned}$$

For the first and second terms in the above equation, we can easily get the projector of the distribution amplitude in the momentum space. However, for the third term we should make some effort to transfer it into the momentum space. By using integration by parts for the third term, after a few steps, (10) can finally be changed to

$$\begin{aligned}&\langle K^-(P)|\bar{s_\alpha}(z)u_\beta(0)|0\rangle\\&=\frac{\mathrm{i}}{\sqrt{2N_c}}\int_0^1 \mathrm{d}x\mathrm{e}^{\mathrm{i}xP\cdot z}\left[\gamma_5\ \not{P}\phi_K(x)+m_0\gamma_5\phi_P(x)\right.\\&\quad\left.+m_0[\gamma_5(\not{n}\ \not{v}-1)]\phi_K^t(x)\right]_{\beta\alpha},\qquad(11)\end{aligned}$$

where $\phi_K^t(x)=\frac{1}{6}\frac{\mathrm{d}}{\mathrm{d}x}\phi_\sigma(x)$, and the vector n is parallel to the K meson momentum p_K. Also, $m_{0K}=m_K^2/(m_u+m_s)$ is a scale characterized by chiral perturbation theory. For the π meson, the corresponding scale is defined by $m_{0\pi}=m_\pi^2/(m_u+m_d)$.

For the light vector meson ρ, ω and K^*, we need to distinguish their longitudinal polarization and transverse polarization. If the K^* meson (as the other vector mesons) is longitudinally polarized, we can write its wave function in longitudinal polarization [8,21]

$$\begin{aligned}&\langle K^{*-}(P,\epsilon_{\mathrm{L}})|\bar{d_\alpha}(z)u_\beta(0)|0\rangle=\frac{1}{\sqrt{2N_c}}\int_0^1\mathrm{d}x\mathrm{e}^{\mathrm{i}xP\cdot z}\qquad(12)\\&\times\left\{\not{\epsilon}\left[\not{p}_{K^*}\phi_{K^*}^t(x)+m_{K^*}\phi_{K^*}(x)\right]+m_{K^*}\phi_{K^*}^s(x)\right\}.\end{aligned}$$

The second term in the above equation is the leading twist wave function (twist-2), while the first and third terms are sub-leading twist (twist-3) wave functions. If the K^* meson is transversely polarized, its wave function is then

$$\begin{aligned}&\langle K^{*-}(P,\epsilon_{\mathrm{T}})|\bar{d_\alpha}(z)u_\beta(0)|0\rangle\\&=\frac{1}{\sqrt{2N_c}}\int_0^1\mathrm{d}x\mathrm{e}^{\mathrm{i}xP\cdot z}\left\{\not{\epsilon}\left[\not{p}_{K^*}\phi_{K^*}^{\mathrm{T}}(x)+m_{K^*}\phi_{K^*}^v(x)\right]\right.\\&\quad\left.+\mathrm{i}m_{K^*}\epsilon_{\mu\nu\rho\sigma}\gamma_5\gamma^\mu\epsilon^\nu n^\rho v^\sigma\phi_{K^*}^a(x)\right\}.\qquad(13)\end{aligned}$$

Here the leading twist wave function for the transversely polarized K^* meson is the first term which is proportional to $\phi_{K^*}^{\mathrm{T}}$.

4 $B\to P$ form factors

The $B\to P$ form factors are defined as follows:

$$\begin{aligned}&\langle P(p_1)|\bar{q}\gamma_\mu b|\bar{B}(p_B)\rangle\\&=\left[(p_B+p_1)_\mu-\frac{m_B^2-m_P^2}{q^2}q_\mu\right]F_1(q^2)\\&\quad+\frac{m_B^2-m_P^2}{q^2}q_\mu F_0(q^2),\qquad(14)\end{aligned}$$

where $q=p_B-p_1$. In order to cancel the poles at $q^2=0$, we must impose the condition

$$F_1(0)=F_0(0).$$

That means in the large-recoil limit, that we need only calculate one independent form factor for the vector current. For the tensor operator, there is also only one independent form factor, which is important for the semi-leptonic decay $B\to K\ell^+\ell^-$:

$$\begin{aligned}\langle P(p_1)|\bar{q}\sigma_{\mu\nu}b|\bar{B}(p_B)\rangle&=\mathrm{i}\left[p_{1\mu}q_\nu-q_\mu p_{1\nu}\right]\\&\quad\times\frac{2F_{\mathrm{T}}(q^2)}{m_B+m_P},\qquad(15)\\\langle P(p_1)|\bar{q}\sigma_{\mu\nu}\gamma_5 b|\bar{B}(p_B)\rangle&=\epsilon_{\mu\nu\alpha\beta}p_1{}^\alpha q^\beta\\&\quad\times\frac{2F_{\mathrm{T}}(q^2)}{m_B+m_P}.\qquad(16)\end{aligned}$$

In the previous section we have discussed the wave functions of the factorization formula in (2). In this section, we will calculate the hard part H. This part involves the current operators and the necessary hard gluon connecting the current operator and the spectator quark. Since the final results are expressed as integrations of the distribution function variables, we will show the whole amplitude for each diagram including wave functions and Sudakov factors.

There are two types of diagrams contributing to the $B\to K$ form factors which are shown in Fig. 1. The sum of their amplitudes is given by

$$\begin{aligned}&F_1(q^2=0)=F_0(q^2=0)\\&=8\pi C_F m_B^2\int_0^1\mathrm{d}x_1\mathrm{d}x_2\int_0^\infty b_1\mathrm{d}b_1b_2\mathrm{d}b_2\\&\times\{h_e(x_1,x_2,b_1,b_2)\left(\phi_B(x_1,b_1)\right.\\&\times\left[(1+x_2)\phi_K^A(x_2,b_2)\right.\\&+\left.r_K(1-2x_2)\left(\phi_K^P(x_2,b_2)+\phi_K^t(x_2,b_2)\right)\right]\\&-\bar{\phi}_B(x_1,b_1)\left[\phi_K^A(x_2,b_2)\right.\\&\quad\left.\left.-\ r_Kx_2\left(\phi_K^P(x_2,b_2)+\phi_K^t(x_2,b_2)\right)\right]\right)\\&\times\alpha_{\mathrm{s}}(t_e^1)\exp[-S_{ab}(t_e^1)]\\&\quad+2r_K\phi_K^P(x_2,b_2)\phi_B(x_1,b_1)\alpha_{\mathrm{s}}(t_e^2)h_e(x_2,x_1,b_2,b_1)\\&\times\ \exp[-S_{ab}(t_e^2)]\}\ ,\qquad(17)\end{aligned}$$

where $r_K=m_{0K}/m_B=m_K^2/[m_B(m_s+m_d)]$; $C_F=4/3$ is a color factor. The function h_e, the scales t_e^i and the Sudakov factors S_{ab} are displayed at the end of this section.

For the $B\to\pi$ form factors, one needs only replace the above K meson distribution amplitudes ϕ_K^i by pion distribution amplitudes ϕ_π^i and replace the scale parameter r_K by $r_\pi=m_{0\pi}/m_B=m_\pi^2/[m_B(m_u+m_d)]$, respectively.

For the tensor operator we get the form factor formulas:

$$\begin{aligned}&F_{\mathrm{T}}(q^2=0)\\&=8\pi C_F m_B^2\int_0^1\mathrm{d}x_1\mathrm{d}x_2\int_0^\infty b_1\mathrm{d}b_1b_2\mathrm{d}b_2\\&\times\{h_e(x_1,x_2,b_1,b_2)\left(\phi_B(x_1,b_1)\right.\\&\times\left[\phi_K^A(x_2,b_2)-x_2r_K\phi_K^P(x_2,b_2)\right.\\&\quad\left.+\ r_K(2+x_2)\phi_K^t(x_2,b_2)\right]-\bar{\phi}_B(x_1,b_1)\\&\times\left.\left[\phi_K^A(x_2,b_2)-r_K\phi_K^P(x_2,b_2)+r_K\phi_K^t(x_2,b_2)\right]\right)\end{aligned}$$

$$\times\ \alpha_s(t_e^1)\exp[-S_{ab}(t_e^1)] + 2r_K h_e(x_2,x_1,b_2,b_1)\alpha_s(t_e^2)\phi_B(x_1,b_1) \times\ \phi_K^P(x_2,b_2)\exp[-S_{ab}(t_e^2)]\Big\}\ . \quad (18)$$

In the above equations, we have used the assumption that $x_1 << x_2$. Since the light quark momentum fraction x_1 in the B meson is peaked at the small region, while the quark momentum fraction x_2 of the K meson is peaked around 0.5, this is not a bad approximation. The numerical results also show that this approximation makes very little difference in the final result. After using this approximation, all the diagrams are functions of $k_1^- = x_1 m_B/\sqrt{2}$ of the B meson only, independent of the variable of k_1^+.

The function h_e, coming from the Fourier transform of the hard amplitude H, is

$$\begin{aligned} h_e(x_1,x_2,b_1,b_2) &= K_0\left(\sqrt{x_1x_2}m_Bb_1\right) \\ &\times [\theta(b_1-b_2)K_0\left(\sqrt{x_2}m_Bb_1\right)I_0\left(\sqrt{x_2}m_Bb_2\right) \\ &+\ \theta(b_2-b_1)K_0\left(\sqrt{x_2}m_Bb_2\right)I_0\left(\sqrt{x_2}m_Bb_1\right)]\,S_t(x_2), \end{aligned} \quad (19)$$

where J_0 is the Bessel function and K_0, I_0 are modified Bessel functions.

The Sudakov factors used in the text are defined as

$$\begin{aligned} S_{ab}(t) &= s\left(x_1m_B/\sqrt{2},b_1\right) + s\left(x_2m_B/\sqrt{2},b_2\right) \\ &+ s\left((1-x_2)m_B/\sqrt{2},b_2\right) \\ &- \frac{1}{\beta_1}\left[\ln\frac{\ln(t/\Lambda)}{-\ln(b_1\Lambda)} + \ln\frac{\ln(t/\Lambda)}{-\ln(b_2\Lambda)}\right], \end{aligned} \quad (20)$$

where the functions $s(q,b)$ are defined in Appendix A of [6]. The hard scales t_i in the above equations are chosen as the largest scale of the virtualities of the internal particles in the hard b quark decay diagrams,

$$\begin{aligned} t_e^1 &= \max(\sqrt{x_2}m_B, 1/b_1, 1/b_2)\ , \\ t_e^2 &= \max(\sqrt{x_1}m_B, 1/b_1, 1/b_2)\ . \end{aligned} \quad (21)$$

5 $B \to V$ form factors

For the $B \to K^*$ form factors, we first define the axial vector current part,

$$\begin{aligned} &\langle K^*(p_1)|\bar{q}\gamma_\mu\gamma_5 b|\bar{B}(p_B)\rangle \\ &= \mathrm{i}\left(\epsilon_\mu^* - \frac{\epsilon^*\cdot q}{q^2}q_\mu\right)(m_B+m_{K^*})A_1(q^2) \\ &- \mathrm{i}\left((p_B+p_1)_\mu - \frac{(m_B^2-m_{K^*}^2)}{q^2}q_\mu\right)(\epsilon^*\cdot q)\frac{A_2(q^2)}{m_B+m_{K^*}} \\ &+ \mathrm{i}\frac{2m_{K^*}(\epsilon^*\cdot q)}{q^2}q_\mu A_0(q^2), \end{aligned} \quad (22)$$

where ϵ^* is the polarization vector of the K^* meson. To cancel the poles at $q^2 = 0$, we must have

$$\begin{aligned} 2m_{K^*}A_0(0) &= (m_B+m_{K^*})A_1(0) \\ &\quad -(m_B-m_{K^*})A_2(0). \end{aligned} \quad (23)$$

For the vector current, only one form factor V is defined:

$$\langle K^*(p_1)|\bar{q}\gamma_\mu b|\bar{B}(p_B)\rangle = \epsilon_{\mu\nu\alpha\beta}\epsilon^{\nu*}p_B^\alpha p_1^\beta\frac{2V(q^2)}{(m_B+m_{K^*})}. \quad (24)$$

For the tensor operators, three form factors are defined:

$$\begin{aligned} &\langle K^*(p_1)|\bar{q}\sigma_{\mu\nu}b|\bar{B}(p_B)\rangle \\ &= -\mathrm{i}\epsilon_{\mu\nu\alpha\beta}\epsilon^{\alpha*}p_1^\beta T_1(q^2) - \mathrm{i}\epsilon_{\mu\nu\alpha\beta}\epsilon^{\alpha*}p_B^\beta T_2(q^2) \\ &\quad -\mathrm{i}T_3(q^2)\frac{(p_B\cdot\epsilon^*)}{p_B\cdot p_1}\epsilon_{\mu\nu\alpha\beta}p_1^\alpha p_B^\beta, \end{aligned} \quad (25)$$

$$\begin{aligned} &\langle K^*(p_1)|\bar{q}\sigma^{\mu\nu}\gamma_5 b|\bar{B}(p_B)\rangle \\ &= \left(p_1^\mu p_B^{*\nu} - p_B^{*\mu}p_1^\nu\right)\frac{(p_B\cdot\epsilon^*)}{p_B\cdot p_1}T_3(q^2) \\ &+ \left(\epsilon^{*\mu}p_B^\nu - p_B^\mu\epsilon^{*\nu}\right)T_2(q^2) \\ &+ \left[\epsilon^{*\mu}p_1^\nu - p_1^\mu\epsilon^{*\nu}\right]T_1(q^2). \end{aligned} \quad (26)$$

Another frequently used set of tensor form factors are defined as below [24]:

$$\begin{aligned} &\langle K^*(p_1)|\bar{q}\sigma_{\mu\nu}q^\nu\frac{(1+\gamma_5)}{2}b|\bar{B}(p_B)\rangle \\ &= 2\mathrm{i}\epsilon_{\mu\nu\alpha\beta}\epsilon^{\nu*}p_B^\alpha p_1^\beta T_1'(q^2) \\ &+ \left[\epsilon_\mu^*(m_B^2-m_{K^*}^2) - (q\cdot\epsilon^*)(p_1+p_B)_\mu\right]T_2'(q^2) \\ &+ (q\cdot\epsilon^*)\left[q_\mu - \frac{q^2}{m_B^2-m_{K^*}^2}(p_1+p_B)_\mu\right]T_3'(q^2). \end{aligned} \quad (27)$$

They are useful for the discussion of the flavor changing neutral current decay $B \to K^*\gamma$ and $B \to K^*\ell^+\ell^-$. The relations between the two set of form factors are

$$T_1'(q^2) = \frac{1}{4}\left[T_1(q^2)+T_2(q^2)\right], \quad (28)$$

$$T_2'(q^2) = \frac{1}{4}\left[T_1(q^2)+T_2(q^2) + \frac{q^2}{m_B^2-m_{K^*}^2}\left(T_2(q^2)-T_1(q^2)\right)\right], \quad (29)$$

$$T_3'(q^2) = \frac{1}{4}\left[T_1(q^2)-T_2(q^2)-\frac{m_B^2-m_{K^*}^2}{p_B\cdot p_1}T_3(q^2)\right]. \quad (30)$$

It is easy to see from the above that in the large-recoil limit $q^2 = 0$, $T_1'(0) = T_2'(0)$.

As for the $B \to \rho$, $B \to \omega$ form factors, the definition is similar to the above, just replacing K^* by ρ and ω respectively.

Calculating the corresponding amplitude for Fig. 1a and b, we get the formulas for the form factors at large recoil:

$$\begin{aligned} A_0(q^2=0) &= 8\pi C_F m_B^2\int_0^1 \mathrm{d}x_1\mathrm{d}x_2\int_0^\infty b_1\mathrm{d}b_1 b_2\mathrm{d}b_2 \\ &\times\ \left\{\alpha_s(t_e^1)\exp[-S_{ab}(t_e^1)]\left[\phi_B(x_1,b_1)\right.\right. \\ &\times\ ((1+x_2)\phi_{K^*}(x_2,b_2) \\ &\quad +(1-2x_2)r_{K^*}\left(\phi_{K^*}^t(x_2,b_2)+\phi_{K^*}^s(x_2,b_2)\right)) \\ &-\ \bar{\phi}_B(x_1,b_1) \\ &\times\ \left.\left(\phi_{K^*}(x_2,b_2) - x_2r_{K^*}\left(\phi_{K^*}^t(x_2,b_2)+\phi_{K^*}^s(x_2,b_2)\right)\right)\right] \\ &\times\ h_e(x_1,x_2,b_1,b_2) \\ &+\ 2r_{K^*}\phi_B(x_1,b_1)\phi_{K^*}^s(x_2,b_2)\alpha_s(t_e^2)h_e(x_2,x_1,b_2,b_1) \\ &\times\ \left.\exp[-S_{ab}(t_e^2)]\right\}\ , \end{aligned} \quad (31)$$

where $r_{K^*} = m_{K^*}/m_B$. The form factor A_0 is the one contributing to the non-leptonic B decays $B \to PV$, where the vector meson is longitudinally polarized. This is shown in the above equation, (31): the formula depends only on the longitudinal wave functions. On the other hand, the form factor A_1 contributing to the $B \to VV$ decays depends only on the transverse wave functions, which is shown below:

$$\begin{aligned}
&A_1(q^2=0) = 8\pi C_F m_B(m_B - m_{K^*}) \\
&\times \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \int_0^\infty b_1 \mathrm{d}b_1 b_2 \mathrm{d}b_2 \\
&\times \left\{h_e(x_1,x_2,b_1,b_2)\exp[-S_{ab}(t_e^1)]\right. \\
&\times \alpha_\mathrm{s}(t_e^1)\left[\phi_B(x_1,b_1)\right. \\
&\times \left(\phi_{K^*}^\mathrm{T}(x_2,b_2) + (2+x_2) r_{K^*}\phi_{K^*}^v(x_2,b_2)\right. \\
&\left.\quad -r_{K^*}x_2\phi_{K^*}^a(x_2,b_2)\right) \\
&-\bar\phi_B(x_1,b_1) \\
&\times \left.\left(\phi_{K^*}^\mathrm{T}(x_2,b_2) + r_{K^*}\phi_{K^*}^v(x_2,b_2) - r_{K^*}\phi_{K^*}^a(x_2,b_2)\right)\right] \\
&+ r_{K^*}\phi_B(x_1,b_1)[\phi_{K^*}^v(x_2,b_2) + \phi_{K^*}^a(x_2,b_2)] \\
&\times \left.\alpha_\mathrm{s}(t_e^2) h_e(x_2,x_1,b_2,b_1)\exp[-S_{ab}(t_e^2)]\right\} . \qquad (32)
\end{aligned}$$

The form factor A_2 can be calculated from (23), using the above (31) and (32) for A_0 and A_1. It depends on both transverse and longitudinal wave functions.

The vector form factor V, depending only on transverse wave functions, is expressed as

$$\begin{aligned}
&V(q^2=0) = 8\pi C_F m_B(m_B + m_{K^*}) \\
&\times \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \int_0^\infty b_1 \mathrm{d}b_1 b_2 \mathrm{d}b_2 \left\{\alpha_\mathrm{s}(t_e^1) h_e(x_1,x_2,b_1,b_2)\right. \\
&\times \left(\phi_B(x_1,b_1)\left[\phi_{K^*}^\mathrm{T}(x_2,b_2) + (2+x_2)r_{K^*}\phi_{K^*}^a(x_2,b_2)\right.\right. \\
&\quad \left. - r_{K^*}x_2\phi_{K^*}^v(x_2,b_2)\right] \\
&\quad -\bar\phi_B(x_1,b_1)\left[\phi_{K^*}^\mathrm{T}(x_2,b_2) + r_{K^*}\phi_{K^*}^a(x_2,b_2)\right. \\
&\quad \left.\left. - r_{K^*}\phi_{K^*}^v(x_2,b_2)\right]\right)\exp[-S_{ab}(t_e^1)] \\
&+ r_{K^*}h_e(x_2,x_1,b_2,b_1)\left[\phi_{K^*}^v(x_2,b_2) + \phi_{K^*}^a(x_2,b_2)\right] \qquad (33) \\
&\times \left.\phi_B(x_1,b_1)\,\alpha_\mathrm{s}(t_e^2)\exp[-S_{ab}(t_e^2)]\right\} .
\end{aligned}$$

As for the tensor form factors, T_1 and T_2, contributing to the $B \to K^*\gamma$ decay, depend only on transverse wave functions:

$$\begin{aligned}
&T_1(q^2=0) = 16\pi C_F m_B^2 \\
&\times \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \int_0^\infty b_1 \mathrm{d}b_1 b_2 \mathrm{d}b_2 \\
&\times \left\{\alpha_\mathrm{s}(t_e^1) h_e(x_1,x_2,b_1,b_2)\left(\phi_B(x_1,b_1)\right.\right. \\
&\times \left[(1+x_2)\phi_{K^*}^\mathrm{T}(x_2,b_2) + 2(1-x_2)r_{K^*}\phi_{K^*}^a(x_2,b_2)\right. \\
&\quad \left. - 2x_2 r_{K^*}\phi_{K^*}^v(x_2,b_2)\right] \\
&- \bar\phi_B(x_1,b_1)\left[\phi_{K^*}^\mathrm{T}(x_2,b_2) + (1-x_2)r_{K^*}\phi_{K^*}^a(x_2,b_2)\right. \\
&\quad \left.\left. - (1+x_2)r_{K^*}\phi_{K^*}^v(x_2,b_2)\right]\right)\exp[-S_{ab}(t_e^1)] \\
&+ r_{K^*}h_e(x_2,x_1,b_2,b_1)\left[\phi_{K^*}^v(x_2,b_2) + \phi_{K^*}^a(x_2,b_2)\right] \qquad (34) \\
&\times \left.\phi_B(x_1,b_1)\alpha_\mathrm{s}(t_e^2)\exp[-S_{ab}(t_e^2)]\right\} .
\end{aligned}$$

$$\begin{aligned}
&T_2(q^2=0) = 16\pi C_F m_B^2 \\
&\times \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \int_0^\infty b_1 \mathrm{d}b_1 b_2 \mathrm{d}b_2\, \alpha_\mathrm{s}(t_e^1) h_e(x_1,x_2,b_1,b_2) r_{K^*} \\
&\times \left[\phi_{K^*}^v(x_2,b_2) - \phi_{K^*}^a(x_2,b_2)\right] \\
&\times \left(\phi_B(x_1,b_1) - \bar\phi_B(x_1,b_1)\right)\exp[-S_{ab}(t_e^1)] , \qquad (35)
\end{aligned}$$

while the form factor T_3 depends on both longitudinal and transverse wave functions:

$$\begin{aligned}
&T_3(q^2=0) = 16\pi C_F m_B^2 \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \int_0^\infty b_1 \mathrm{d}b_1 b_2 \mathrm{d}b_2 \\
&\times \left\{\alpha_\mathrm{s}(t_e^1) h_e(x_1,x_2,b_1,b_2)\left(\phi_B(x_1,b_1)\right.\right. \\
&\times \left[\phi_{K^*}(x_2,b_2) + (2+x_2)r_{K^*}\phi_{K^*}^t(x_2,b_2)\right. \\
&\quad \left. -x_2 r_{K^*}\phi_{K^*}^s(x_2,b_2)\right] \\
&- \bar\phi_B(x_1,b_1)\left[\phi_{K^*}(x_2,b_2) + r_{K^*}\phi_{K^*}^t(x_2,b_2)\right. \\
&\quad \left.\left. -r_{K^*}\phi_{K^*}^s(x_2,b_2)\right]\right)\exp[-S_{ab}(t_e^1)] \\
&+ 2r_{K^*}h_e(x_2,x_1,b_2,b_1)\phi_{K^*}^s(x_2,b_2) \qquad (36) \\
&\times \phi_B(x_1,b_1)\alpha_\mathrm{s}(t_e^2) \\
&\times \left.\exp[-S_{ab}(t_e^2)]\right\} r_{K^*} - 2r_{K^*}^2 T_1 - T_2 .
\end{aligned}$$

6 Numerical calculations and discussion

In the numerical calculations we use

$$\begin{aligned}
&\Lambda_{\overline{\mathrm{MS}}}^{(f=4)} = 250\,\mathrm{MeV},\ f_\pi = 132\,\mathrm{MeV},\ f_K = 160\,\mathrm{MeV}, \\
&f_B = 190\,\mathrm{MeV},\ m_{0\pi} = 1.4\,\mathrm{GeV},\ m_{0K} = 1.7\,\mathrm{GeV}, \\
&M_B = 5.2792\,\mathrm{GeV},\ f_{K^*} = 220\,\mathrm{MeV},\ f_{K^*}^\mathrm{T} = 180\,\mathrm{MeV}, \\
&M_W = 80.41\,\mathrm{GeV},\ f_\rho = 217\,\mathrm{MeV},\ f_\rho^\mathrm{T} = 160\,\mathrm{MeV}, \\
&f_\omega = 195\,\mathrm{MeV},\ f_\omega^\mathrm{T} = 160\mathrm{MeV}. \qquad (37)
\end{aligned}$$

The distribution amplitudes $\phi_\pi^i(x)$, $\phi_K^i(x)$, $\phi_\rho^i(x)$ ($\phi_\omega^i(x)$) and $\phi_{K^*}^i(x)$ of the light mesons used in the numerical calculation are listed in Appendix A.

Figure 2a displays the contributions to the $B \to \pi$ transition form factor at the large-recoil limit $q^2 \to 0$ from different ranges of α_s/π, where the hard scale t is chosen as (21), i.e., the maximum virtuality of both internal quarks and gluons in the hard b quark decay diagrams. It shows that most of the contribution comes from the range $\alpha_\mathrm{s}/\pi < 0.3$, implying that the average scale is around $\sqrt{\Lambda_\mathrm{QCD} m_B}$. For other B to light meson transition form factor calculations, we have very similar results. It is observed that with the hard scale chosen in (21), PQCD is applicable to $B \to$ light meson transition form factors. A recent study shows that PQCD is even applicable to $B \to D^{(*)}$ form factors [9]. However, a different perturbative percentage distribution over α_s/π was observed in [19, 22]. We check the reason which causes this difference and find that the most important reason is the way of choosing the hard scale t. If the hard scale is chosen as the maximum virtuality of only the gluon and other transverse momentum scales, i.e., $t \equiv \max(\sqrt{x_1 x_2} m_B, 1/b_1, 1/b_2)$, the perturbative percentage distribution will be similar to the one in [19,22], as shown in Fig. 2b. Therefore the way of

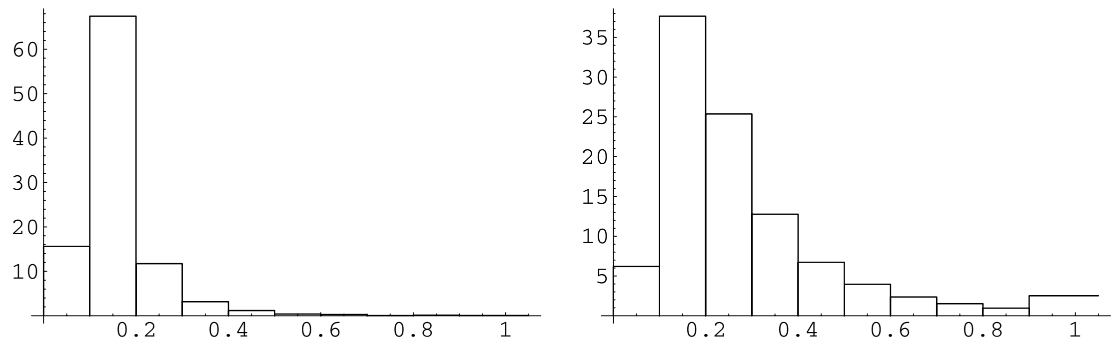

Fig. 2a,b. Contributions to the $B \to \pi$ transition form factors ($F^{B\pi}(0)$) from different ranges of α_s/π, **a** with the hard scale chosen as virtualities of internal particles including both quarks and gluons; **b** the hard scale chosen as virtualities of only internal gluons

Table 1. B meson transition form factors at $q^2 = 0$ with the hard scale chosen in (21), and the numbers in parentheses are results without the contribution of $\bar{\phi}_B$

Process	$F_0(0) = F_1(0)$	$F_T(0)$					
$B \to \pi$	0.292 ± 0.030	0.278 ± 0.028					
	(0.199)	(0.189)					
$B \to K$	0.321 ± 0.036	0.311 ± 0.033					
	(0.231)	(0.223)					
Process	$V(0)$	$A_0(0)$	$A_1(0)$	$A_2(0)$	$T_1(0)$	$T_2(0)$	$T_3(0)$
$B \to \rho$	0.318 ± 0.032	0.366 ± 0.036	0.25 ± 0.02	0.21 ± 0.01	0.56 ± 0.05	0.013 ± 0.001	0.06 ± 0.01
	(0.226)	(0.256)	(0.17)	(0.14)	(0.41)	(0.004)	(0.05)
$B \to \omega$	0.305 ± 0.030	0.347 ± 0.036	0.24 ± 0.02	0.20 ± 0.02	0.53 ± 0.05	0.012 ± 0.001	0.06 ± 0.01
	(0.212)	(0.250)	(0.16)	(0.13)	(0.38)	(0.003)	(0.05)
$B \to K^*$	0.406 ± 0.042	0.455 ± 0.047	0.30 ± 0.03	0.24 ± 0.02	0.69 ± 0.08	0.007 ± 0.001	0.09 ± 0.01
	(0.293)	(0.336)	(0.21)	(0.16)	(0.51)	(-0.001)	(0.07)

choosing the hard scale is one of the important ingredients in the PQCD approach, which deserves more concern.[1] Provided that the virtuality of the internal quark momentum (longitudinal) must appear as a characteristic scale in the hard diagram, in general it should be taken into account. Therefore we think that it is reasonable to choose both the virtualities of internal quarks and gluons as the hard scale. Certainly the most powerful proof of this point should be performed under the help of a numerical calculation of higher order loop corrections. However, such a deeper discussion of this problem is beyond the scope of this paper; it shall be left to other attempts.

The results of the $B \to P$, V light meson transition form factors are given in Table 1 with the hard scale chosen in (21). Compared with previous PQCD calculations on some $B \to P$, V transition form factors [8,9,23], the current work is different from them mainly on two points: (1) The B meson wave function used here is the one derived from the equation of motion in heavy quark effective theory [15]. There is only one free parameter in the functions of the distribution amplitudes, $\bar{\Lambda}$. We show the results for $\bar{\Lambda} = (700 \pm 50)$ MeV in Table 1. All the form factors are sensitive to this parameter, i.e. sensitive to the shape of the B meson distribution amplitudes.
(2) Two Lorentz structure terms of B meson wave function, both ϕ_B and $\bar{\phi}_B$ defined in (4) and (6), are taken into account in this work. To see how much the $\bar{\phi}_B$ term contributes, we give the results without the contribution of $\bar{\phi}_B$ in the parentheses in Table 1. They show that the contribution of $\bar{\phi}_B$ is about 30%. The dominant contribution comes from the ϕ_B term. This result shows that simply dropping the contribution of $\bar{\phi}_B$ is not a good approximation.

We compare some of the results calculated in this work with lattice calculation by the UKQCD collaboration [25] in Table 2. It shows that our results are consistent with theirs.

The $B \to K^*$ form factors are useful for the calculation of the flavor changing neutral current process $B \to K^*\gamma$ and $B \to K^*\ell^+\ell^-$, which have been discussed many times [26]. We show some of them in Table 3 for comparison. It

[1] By numerical check we find that these two different choices of the hard scale only slightly affect the magnitude of the form factors. For example it can only change the $B \to \pi$ from factor by a few percent.

Table 2. Form factors at $q^2 = 0$ for $B \to \pi$ and $B \to \rho$ transitions calculated in this work and UKQCD

	$F_0(0) = F_1(0)$	$V(0)$	$A_0(0)$	$A_1(0)$	$A_2(0)$
UKQCD [25]	0.27 ± 0.11	$0.35^{+0.06}_{-0.05}$	$0.30^{+0.06}_{-0.04}$	$0.27^{+0.05}_{-0.04}$	$0.26^{+0.05}_{-0.03}$
This work	0.292	0.318	0.366	0.250	0.210

Table 3. Some form factors at $q^2 = 0$ for $B \to K^*$ transitions calculated in this work and some other works

	$T_1'(0) = T_2'(0)$	$A_0(0)$	$A_1(0)$
Quark model [27]	0.155	0.32	0.26
QCD sum rule [24]	0.19 ± 0.03	0.3 ± 0.03	0.37 ± 0.03
Light-cone sum rule [28]	0.18	0.27	0.36
Lattice QCD [25]	$0.16^{+0.02}_{-0.01}$	0.33	0.29
Dispersion quark model [29]	0.177	0.44	0.33
This work	0.175	0.455	0.297

is easy to see that our results agree with the lattice calculations [25] and the results calculated using the lattice-constrained dispersion quark model [29].

7 Summary

We have calculated $B \to P$ and $B \to V$ transition form factors in the PQCD approach. We not only calculated the B to light meson transition form factors defined in vector and axial vector currents, but also the form factors defined in tensor currents $\bar{q}\sigma_{\mu\nu}b$ and $\bar{q}\sigma_{\mu\nu}\gamma_5 b$, which can be used to study semi-leptonic and radiative B decays induced by magnetic penguin operators $\bar{q}\sigma_{\mu\nu}(1+\gamma_5)bF_{\mu\nu}$. With the hard scale appropriately chosen, Sudakov effects can effectively suppress the long-distance dynamics, which makes the short-distance contribution dominate. The characteristic scale in B to light meson transition processes is around $\sqrt{\Lambda_{\rm QCD} m_B}$.

We considered both of the two Lorentz structures of the B meson wave functions, and found that the contribution of $\bar{\phi}_B$ defined in (4) and (6) is about 30%.

Finally we compared our results with a lattice calculation, some quark model and QCD sum rule calculations; we found that they are consistent with our results.

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A Wave functions of light mesons used in the numerical calculation

For the light meson wave function, we neglect the b dependence part, which is not important in the numerical analysis.

The distribution amplitude ϕ_π^A for the twist-2 wave function and the distribution amplitudes ϕ_π^P and ϕ_π^t of the twist-3 wave functions are taken from [20]:

$$\phi_\pi^A(x) = \frac{3f_\pi}{\sqrt{6}} x(1-x) \times \left[1 + 0.44 C_2^{3/2}(t) + 0.25 C_4^{3/2}(t)\right], \quad (38)$$

$$\phi_\pi^P(x) = \frac{f_\pi}{2\sqrt{6}} \left[1 + 0.43 C_2^{1/2}(t) + 0.09 C_4^{1/2}(t)\right], \quad (39)$$

$$\phi_\pi^t(x) = \frac{f_\pi}{2\sqrt{6}}\, t\left[1 + 0.55(10x^2 - 10x + 1)\right], \quad (40)$$

where $t = 1-2x$. The Gegenbauer polynomials are defined by

$$C_2^{1/2}(t) = \frac{1}{2}(3t^2-1),\ C_4^{1/2}(t) = \frac{1}{8}(35t^4 - 30t^2 + 3),$$
$$C_2^{3/2}(t) = \frac{3}{2}(5t^2-1),\ C_4^{3/2}(t) = \frac{15}{8}(21t^4 - 14t^2 + 1), \quad (41)$$

whose coefficients correspond to $m_{0\pi} = 1.4\,\mathrm{GeV}$.

We choose the different distribution amplitudes of the ρ meson's longitudinal wave function as [21]

$$\phi_\rho(x) = \frac{3f_\rho}{\sqrt{6}} x(1-x)\left[1 + 0.18 C_2^{3/2}(t)\right], \quad (42)$$

$$\phi_\rho^t(x) = \frac{f_\rho^{\rm T}}{2\sqrt{6}} \left\{3t^2 + 0.3t^2\left[5t^2 - 3\right] + 0.21\left[3 - 30t^2 + 35t^4\right]\right\}, \quad (43)$$

$$\phi_\rho^s(x) = \frac{3f_\rho^{\rm T}}{2\sqrt{6}}\, t\left[1 + 0.76(10x^2 - 10x + 1)\right]. \quad (44)$$

For the transverse ρ meson we use [21]

$$\phi_\rho^{\rm T}(x) = \frac{3f_\rho^{\rm T}}{\sqrt{6}} x(1-x)\left[1 + 0.2 C_2^{3/2}(t)\right], \quad (45)$$

$$\phi_\rho^v(x) = \frac{f_\rho}{2\sqrt{6}} \left\{ \frac{3}{4}(1+t^2) + 0.24(3t^2-1) + 0.12(3-30t^2+35t^4) \right\}, \quad (46)$$

$$\phi_\rho^a(x) = \frac{3f_\rho}{4\sqrt{6}}\, t\left[1+0.93(10x^2-10x+1)\right]. \quad (47)$$

For the ω meson, we use the same as the above ρ meson, except exchanging the decay constant f_ρ with f_ω.

We use ϕ_K^A of the K meson twist-2 wave function and ϕ_K^P and ϕ_K^t of the twist-3 wave functions from [20,21,30]:

$$\phi_K^A(x) = \frac{3f_K}{\sqrt{6}} x(1-x)\left[1+0.51t+0.3\{5t^2-1\}\right], \quad (48)$$

$$\phi_K^P(x) = \frac{f_K}{2\sqrt{6}}\left[1+0.12(3t^2-1) - 0.12(3-30t^2+35t^4)/8\right], \quad (49)$$

$$\phi_K^t(x) = \frac{f_K}{2\sqrt{6}}\, t\left[1+0.35(10x^2-10x+1)\right], \quad (50)$$

whose coefficients correspond to $m_{0K} = 1.7\,\mathrm{GeV}$.

We choose the light-cone distribution amplitudes of the K^* meson longitudinal wave function as [21]

$$\phi_{K^*}(x) = \frac{3f_{K^*}}{\sqrt{6}} x(1-x)\left[1+0.57t+0.07C_2^{3/2}(t)\right], \quad (51)$$

$$\phi_{K^*}^t(x) = \frac{f_{K^*}^{\mathrm{T}}}{2\sqrt{6}}\Big\{0.3t(3t^2+10t-1)+1.68C_4^{1/2}(t) + 0.06t^2(5t^2-3)+0.36[1-2t-2t\ln(1-x)]\Big\}, \quad (52)$$

$$\phi_{K^*}^s(x) = \frac{f_{K^*}^{\mathrm{T}}}{2\sqrt{6}}\left\{3t\left[1+0.2t+0.6(10x^2-10x+1)\right] - 0.12x(1-x)+0.36[1-6x-2\ln(1-x)]\right\}. \quad (53)$$

The following light-cone distribution amplitudes of the K^* transverse wave function are used:

$$\phi_{K^*}^{\mathrm{T}}(x) = \frac{3f_{K^*}^{\mathrm{T}}}{\sqrt{6}} x(1-x)\left[1+0.6t+0.04C_2^{3/2}(t)\right], \quad (54)$$

$$\phi_{K^*}^v(x) = \frac{f_{K^*}}{2\sqrt{6}}\left\{\frac{3}{4}(1+t^2+0.44t^3)+0.4C_2^{1/2}(t) + 0.88C_4^{1/2}(t)+0.48[2x+\ln(1-x)]\right\}, \quad (55)$$

$$\phi_{K^*}^a(x) = \frac{f_{K^*}}{4\sqrt{6}}\left\{3t\left[1+0.19t+0.81(10x^2-10x+1)\right] - 1.14x(1-x) + 0.48[1-6x-2\ln(1-x)]\right\}. \quad (56)$$

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